Eco 4306 Economic and Business Forecasting

Lecture 23
Chapter 12: Forecasting the Long Term and the Short Term Jointly

Motivation

- vector autoregressive models (VAR) are a useful tool to forecast stationary time series
- these models are not suitable for time series that contain a unit root and thus are not stationary
- we will next develop a new framework, **vector error correction models (VEC)**, specifically designed to model and forecast time series that contain a unit root

Motivating Example 1

- in equilibrium, demand for goods and services is equal to their supply
- let Y be the production and C the demand
- economy is in equilibrium when $Y = C$
- and as long as the growth of production is equal to the growth of demand $Y = C$ will continue to hold even as Y and C grow over time

Motivating Example 1

- suppose that some shock, e.g. weather reduces level of production, so that $Y < C$
- corrective forces in the economy: a new equilibrium will be achieved - excess demand will be corrected either by decreasing demand or increasing production or a combination of both
- in particular, in the short term, rising prices will partly correct the excess demand, in a longer term production will gradually increase to restore the original equilibrium
- over several periods equilibrium will thus be reinstated by a combination of supply $\Delta Y > 0$ and demand $\Delta C < 0$ adjustments
- similar arguments can be made if the disequilibrium is an excess supply, $Y > C$: in the short run, a reduction in production $\Delta Y < 0$ and an increase in demand $\Delta C > 0$ will remove the excess supply, pushing the economy back toward the long term equilibrium path $Y = C$

Motivating Example 2

- personal consumption expenditures C and personal disposable income I also likely to grow along an equilibrium path
- in short run, households can use their savings or borrow to keep consumption temporarily above the disposable income
- but this is not sustainable in the long run; thus in the long run the marginal propensity to consume $mpc = \frac{C}{I}$ can not grow or decline indefinitely
- in other words, $\log\left(\frac{C}{I}\right) = \log C - \log I$ will be bounded and will tend to self-correct

12.1 Finding a Long-Term Equilibrium Relationship

- identifying long run equilibrium relationships in the data is complicated by the fact that most of the macroeconomic time series (gross domestic product, consumption, price indexes, interest rates, stock prices, exchange rates, ...) have a unit root
- when Y_t and X_t are nonstationary processes (both have a unit root) running a regression

$$Y_t = \beta_0 + \beta_1 X_t + z_t$$

  can lead to **spurious regression problem**

12.1 Finding a Long-Term Equilibrium Relationship

- example of a spurious regression: consider to unrelated random walks $X_t = X_{t-1} + \varepsilon_{x,t}$ and $Y_t = Y_{t-1} + \varepsilon_{y,t}$
- since X_t and Y_t are unrelated, we would hope that estimating the regression

$$Y_t = \beta_0 + \beta_1 X_t + z_t$$

  should yield $\beta_1 \neq 0$
- but this is not going to be the case in general - nonstationarity of X_t and Y_t will lead to a statistically significant β_1

12.1 Finding a Long-Term Equilibrium Relationship

- example of a spurious regression: consider to unrelated random walks $X_t = X_{t-1} + \varepsilon_{x,t}$ and $Y_t = Y_{t-1} + \varepsilon_{y,t}$
- coefficient β_1 is highly statistically significant, its p-value is 0.0004

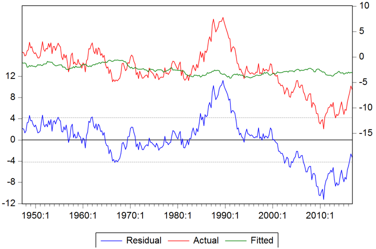
Dependent Variable: Y
Method: Least Squares
Date: 04/30/17 Time: 11:32
Sample: 1947Q2 2016Q4
Included observations: 279

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.171031	0.454560	-2.576186	0.0105
X	0.176082	0.048751	3.611833	0.0004

R-squared	0.044977	Mean dependent var	-2.540054
Adjusted R-squared	0.041529	S.D. dependent var	4.280819
S.E. of regression	4.190987	Akaike info criterion	5.710892
Sum squared resid	4865.330	Schwarz criterion	5.736922
Log likelihood	-794.6694	Hannan-Quinn criter.	5.721334
F-statistic	13.04534	Durbin-Watson stat	0.057505
Prob(F-statistic)	0.000361		

12.1 Finding a Long-Term Equilibrium Relationship

- one hint that the regression is spurious: residuals will show time dependence, first lag in PAC will be close to 1 since residuals are non-stationary
- non-stationary behavior of residuals is clearly visible also in the residuals plot

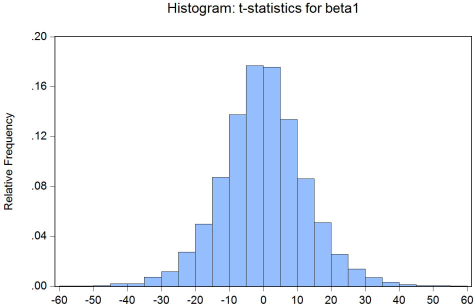


Date: 04/30/17 Time: 11:32
Sample: 1947Q2 2016Q4
Included observations: 279

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.970	0.970	265.15	0.000
		2	0.946	0.095	518.36	0.000
		3	0.921	-0.018	759.32	0.000
		4	0.897	-0.008	988.53	0.000
		5	0.871	-0.029	1205.7	0.000
		6	0.843	-0.059	1410.0	0.000
		7	0.812	-0.088	1599.9	0.000
		8	0.778	-0.074	1774.8	0.000
		9	0.748	0.054	1937.4	0.000
		10	0.724	0.095	2090.4	0.000
		11	0.693	-0.128	2230.7	0.000
		12	0.659	-0.077	2358.1	0.000

12.1 Finding a Long-Term Equilibrium Relationship

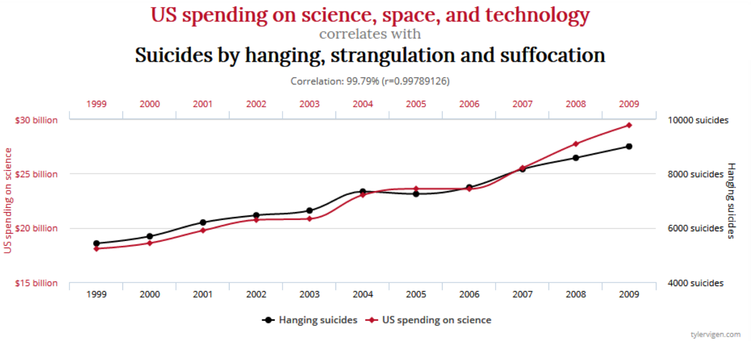
- the above results are not just a coincidence
- suppose that we simulate the two random walks X_t and Y_t 10,000 times and each time run a regressions $Y_t = \beta_0 + \beta_1 X_t + \varepsilon_t$
- the histogram for t-statistics for β_1 is shown below - note that it exceeds ± 2 far more than 5% of times
- in fact, 85.2% of simulations result with t-statistics that exceeds ± 2, making β_1 statistically significant at 5% level



12.1 Finding a Long-Term Equilibrium Relationship

spurious regression problem bottom line

- regression reveals correlation between variables
- but correlation does not imply causation
- and especially when time series are nonstationary or trending, regression results can be meaningless due to spurious correlation
- http://www.tylervigen.com/spurious-correlations



12.1 Finding a Long-Term Equilibrium Relationship

- if the linear regression
$$Y_t = \beta_0 + \beta_1 X_t + z_t$$
is in fact a long-term equilibrium relation, Y_t and X_t are tied to each other in such a way that one cannot wander indefinitely far apart from the other
- in other words, $z_t = Y_t - \beta_1 X_t - \beta_0$, must be a stationary process
- this is the main idea behind the concept of **cointegration**
- two unit root processes, Y_t and X_t, are said to be **cointegrated** if there exist β_0, β_1 such that the linear combination of these two processes $z_t = Y_t - \beta_1 X_t - \beta_0$ is stationary
- this can be generalized further:

  if series $Y_t, X_{1,t}, \ldots, X_{n,t}$ are $I(d)$ and there exists $\beta_0, \beta_1, \ldots, \beta_n$ such that $z_t = Y_t - \beta_1 X_{1,t} - \ldots - \beta_n X_{n,t} - \beta_0$ is $I(d-b)$ then $Y_t, X_{1,t}, \ldots, X_{n,t}$ are said to be cointegrated or order d, b, usually denoted by $CI(d,b)$

12.1 Finding a Long-Term Equilibrium Relationship

- with n variables, there can be potentially up to $n-1$ long run equilibrium cointegrating relationships
- there are two tests used for cointegration testing
- for $r = 0, 1, \dots, n-1$, to test H_0: of r cointegrating relationships against H_A: of more than r cointegrating relationships we use **trace statistic**
- for $r = 0, 1, \dots, n-1$, to test H_0: of r cointegrating relationships against H_A: of $r+1$ cointegrating relationships we use **maximum eigenvalue statistic**
- results of trace and max eigenvalue test may be contradictory; if that happens max eigenvalue test is usually prioritized

12.1 Finding a Long-Term Equilibrium Relationship

- cointegrating relationship can contain a constant, a time trend, or neither
- the long run equilibrium between two variables can thus be

$$Y_t = \beta_1 X_t + z_t$$

or

$$Y_t = \beta_0 + \beta_1 X_t + z_t$$

or

$$Y_t = \beta_0 + \beta_1 X_t + \delta t + z_t$$

12.1 Finding a Long-Term Equilibrium Relationship

- when testing for cointegration in economic, finance, or business time series data, the following four specification of the deterministic components are relevant:

  assuming no deterministic trend in data

  - Case 1: No intercept or trend in CE or test VAR
  - Case 2: Intercept (no trend) in CE, no intercept or trend in VAR

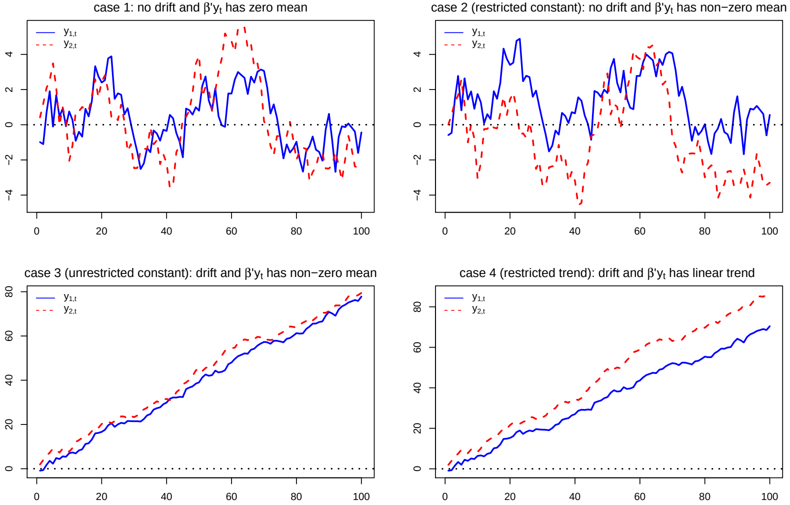
  allowing for a linear deterministic trend in data

  - Case 3: Intercept (no trend) in CE and test VAR
  - Case 4: Intercept and trend in CE, no intercept or trend in VAR

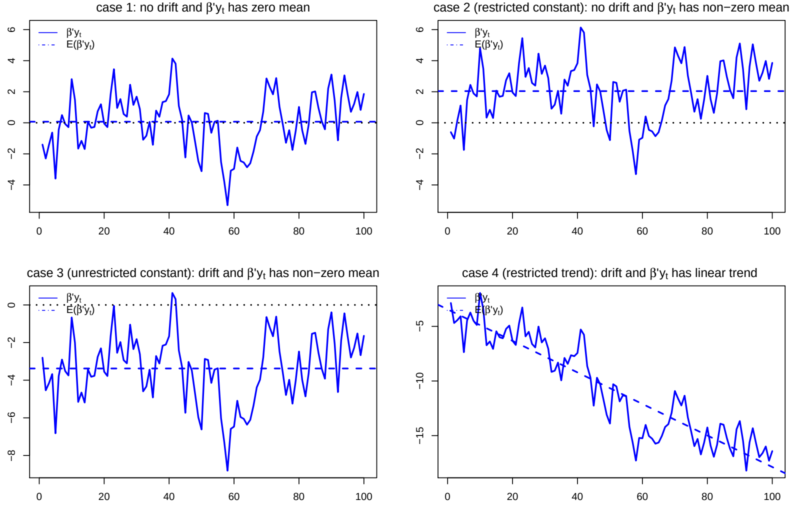
12.1 Finding a Long-Term Equilibrium Relationship

- as a rough rule of thumb
  - when *all* time series in $\boldsymbol{y}_t$ are non-trending like interest rates, exchange rates, inflation rate, unemployment rate, various growth rates, we use Case 2
  - when one or more time series in $\boldsymbol{y}_t$ are trending, e.g. asset prices, macroeconomic aggregates like GDP, consumption, exports, industrial production, employment, national debt, M2 money stock, we start with Case 4 or Case 3, and can consider Case 2 as an alternative if it does not change the results of cointegration test much
- figures on the next two slides show a typical behavior of two cointegrated series under these four cases

12.1 Finding a Long-Term Equilibrium Relationship

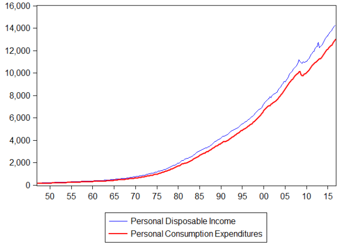


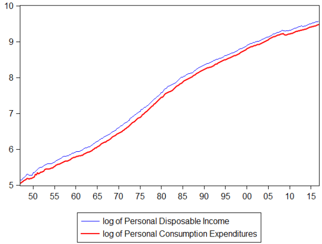
12.1 Finding a Long-Term Equilibrium Relationship



Application 1: Consumption Expenditures and Disposable Income

- figure below shows the quarterly time series for personal consumption expenditures and personal disposable income, npce_q.wf1
- the log transformed series grow over time, they appear to move together, and the gap does not appear to be getting larger
- the figure is thus similar to Case 3 should be considered for the cointegration test





Application 1: Consumption Expenditures and Disposable Income

- to perform the cointegration test in EViews, we first need to the number of lags that should be included in the cointegration test
- to do this first select **Object** → **New Object** → **VAR**, leave "VAR Type" option at Unrestricted VAR, enter `log(nPCE) log(nPDI)` in the "Endogenous Variables" box and `1955Q1 2010Q4` in the "Estimation Sample" box
- then, select **View** → **Lag Structure** → **Lag Length Criteria**

VAR Lag Order Selection Criteria
Endogenous variables: LOG(NPCE) LOG(NPDI)
Exogenous variables: C
Date: 04/05/18 Time: 19:18
Sample: 1955Q1 2010Q4
Included observations: 224

Lag	LogL	LR	FPE	AIC	SC	HQ
0	226.2597	NA	0.000463	-2.002319	-1.971858	-1.990023
1	1568.284	2648.101	3.00e-09	-13.94896	-13.85758	-13.91208
2	1602.522	66.94826	2.29e-09	-14.21895	-14.06664*	-14.15747*
3	1607.747	10.12242	2.27e-09	-14.22988	-14.01665	-14.14381
4	1614.532	13.02516	2.21e-09	-14.25475	-13.98060	-14.14409
5	1619.653	9.738122*	2.19e-09*	-14.26475*	-13.92968	-14.12950
6	1620.323	1.263881	2.25e-09	-14.23503	-13.83904	-14.07519
7	1624.497	7.788911	2.25e-09	-14.23658	-13.77967	-14.05215
8	1627.242	5.073359	2.28e-09	-14.22538	-13.70754	-14.01635

* indicates lag order selected by the criterion

- Akaike criterion (AIC) suggests 5 lags, Schwarz criterion (SC) suggests 2 lags

Application 1: Consumption Expenditures and Disposable Income

- thus, to perform the cointegration test select **View** $\rightarrow$ **Cointegration Test** and in "Lag intervals" box enter 1 5 based on AIC lag length; since both log transformed series are growing, but the gap between them is not getting smaller or larger, select Case 3 Intercept (no trend) in CE and test VAR

Sample: 1955Q1 2010Q4
Included observations: 224
Trend assumption: Linear deterministic trend
Series: LOG(NPCE) LOG(NPDI)
Lags interval (in first differences): 1 to 5

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.096758	26.60375	15.49471	0.0007
At most 1	0.016858	3.808473	3.841466	0.0510

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

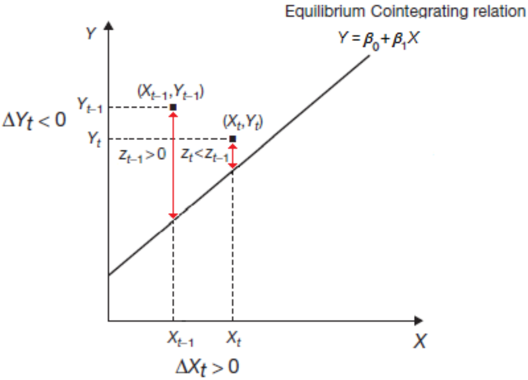
Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.096758	22.79528	14.26460	0.0018
At most 1	0.016858	3.808473	3.841466	0.0510

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

- both trace and max eigenvalue test reject the hypothesis of 0 cointegrating relationships and do not reject the hypothesis of 1 cointegrating relationship

12.2 Quantifying Short-Term Dynamics: Vector Error Correction Model

- consider two cointegrated processes Y_t and X_t, with cointegrating relation $Y = \beta_0 + \beta_1 X$
- suppose that at time $t-1$ the value of the processes are (X_{t-1}, Y_{t-1}) and the system is out of equilibrium with $z_{t-1} = Y_{t-1} - \beta_1 X_{t-1} - \beta_0 > 0$



12.2 Quantifying Short-Term Dynamics: Vector Error Correction Model

how the system will move from $t-1$ to t:

- cointegrating relation exercises a “gravitational pull”, so the system will partially correct the disequilibrium of period $t-1$ and will move toward the equilibrium path
- to reach a new point (X_t, Y_t) from (X_{t-1}, Y_{t-1}), X has increased $\Delta X_t > 0$, and Y has decreased $\Delta Y_t < 0$
- note that there is still a disequilibrium z_t, but of smaller magnitude, $|z_{t-1}| > |z_t|$
- the system has thus partially corrected itself from $t-1$ to t
- if there are no other shocks in the following periods, the system will keep correcting the disequilibrium error until it reaches the equilibrium path, and once there, it will not have any incentive to move out

12.2 Quantifying Short-Term Dynamics: Vector Error Correction Model

- the above verbal description of the dynamics of X_t and Y_t as they move back to the equilibrium is the main idea behind the **vector error correction model (VEC)**
- the short-term dynamics of a simple bivariate VEC are

$$\Delta Y_t = \gamma_1 z_{t-1} + \varepsilon_{1,t}$$
$$\Delta X_t = \gamma_2 z_{t-1} + \varepsilon_{2,t}$$

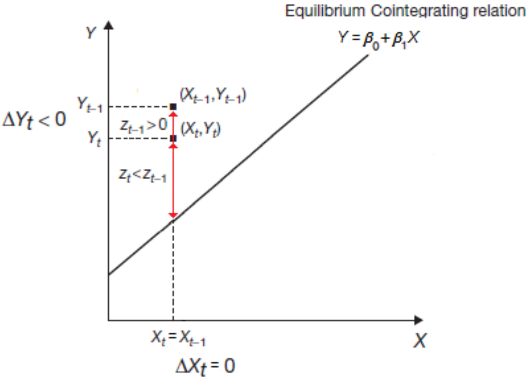
  where again $z_{t-1} = Y_{t-1} - \beta_1 X_{t-1} - \beta_0$, so that the model can be also written as

$$\Delta Y_t = \gamma_1 (Y_{t-1} - \beta_1 X_{t-1} - \beta_0) + \varepsilon_{1,t}$$
$$\Delta X_t = \gamma_2 (Y_{t-1} - \beta_1 X_{t-1} - \beta_0) + \varepsilon_{2,t}$$

- coefficients γ_1 and γ_2 are the **adjustment coefficients**, and indicate how much of the previous disequilibrium error is corrected on moving from $t-1$ to t
- conditions for an error correction model to be stable, so that the error term z_t does not explode but is mean reverting: $\gamma_1 \leq 0$ and $\gamma_2 \geq 0$
- an error correction model must have at least one adjustment coefficient different from zero, $\gamma_1 \neq 0$ and/or $\gamma_2 \neq 0$

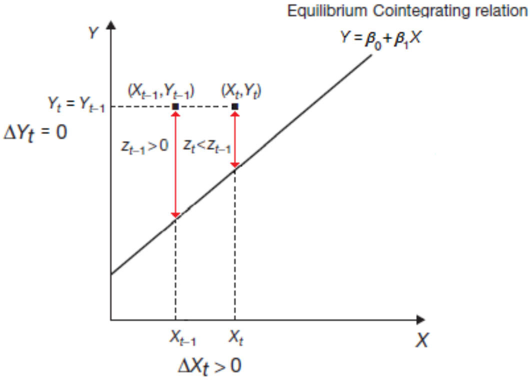
12.2 Quantifying Short-Term Dynamics: Vector Error Correction Model

- if $\gamma_1 < 0$ and $\gamma_2 = 0$ adjustment only takes place in Y,and X remains the same
- for example, if X is income and Y consumption expenditures, this would mean that consumption drops over time if it is unsustainably high, and income remains same over time



12.2 Quantifying Short-Term Dynamics: Vector Error Correction Model

- if $\gamma_1 = 0$ and $\gamma_2 > 0$ adjustment only takes place in X, and Y remains the same
- for example, if X is income and Y consumption expenditures, this would mean that consumption is too high, it will remain unchanged, but income will grow over time



12.2 Quantifying Short-Term Dynamics: Vector Error Correction Model

- VEC model can be expanded to account for two additional features:
  1. ΔY_t and ΔX_t may be autocorrelated
  2. ΔY_t and ΔX_t may be cross-correlated
- thuis is achieved by adding lags of ΔY_t and ΔX_t to explanatory variables
- a vector error correction model or order 1 for two variables is then

$$\Delta Y_t = \gamma_1 z_{t-1} + \kappa_{11} \Delta X_{t-1} + \phi_{11} \Delta Y_{t-1} + \varepsilon_{1,t}$$
$$\Delta X_t = \gamma_2 z_{t-1} + \kappa_{21} \Delta X_{t-1} + \phi_{21} \Delta Y_{t-1} + \varepsilon_{2,t}$$

  and a general vector error correction model or order p for two variables is thus

$$\Delta Y_t = \gamma_1 z_{t-1} + \kappa_{11} \Delta X_{t-1} + \ldots + \kappa_{1p} \Delta X_{t-p} + \phi_{11} \Delta Y_{t-1} + \ldots + \phi_{1p} \Delta Y_{t-p} + \varepsilon_{1,t}$$
$$\Delta X_t = \gamma_2 z_{t-1} + \kappa_{21} \Delta X_{t-1} + \ldots + \kappa_{2p} \Delta X_{t-p} + \phi_{21} \Delta Y_{t-1} + \ldots + \phi_{2p} \Delta Y_{t-p} + \varepsilon_{2,t}$$

- note that this system is very similar to a VAR model with an extra term - the error correction term z_{t-1}
- information criteria, AIC and SIC, are used to select the optimal number of lags p

Application 1: Consumption Expenditures and Disposable Income

- to estimate the VEC model, for time series which are cointegrated, after performing the cointegration test proceed as if you wanted to estimate a VAR, but in the dialog window instead of selecting "Unrestricted VAR" select "Vector Error Correction"
- make sure to select the same deterministic trend specification as before in the cointegration test in the "Cointegration" tab
- the estimated long run relationship is $\log C_t = 0.212 + 1.01 \log I_t$ and the adjustment parameters are $\gamma_1 = -0.143$ and $\gamma_2 = -0.046$

Vector Error Correction Estimates
Date: 04/05/18 Time: 19:18
Sample: 1955Q1 2010Q4
Included observations: 224
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
LOG(NPCE(-1))	1.000000	
LOG(NPDI(-1))	-1.011441 (0.00261) [-387.105]	
C	0.212144	
Error Correction:	**D(LOG(NPCE))**	**D(LOG(NPDI))**
CointEq1	-0.143892 (0.03089) [-4.65889]	-0.046708 (0.03720) [-1.25568]

Application 1: Consumption Expenditures and Disposable Income

- recall that for the long run relationship $Y_t = \beta_0 + \beta_1 X_t$ to be stable and self correcting, $\gamma_1 \leq 0$ and $\gamma_2 \geq 0$ have to be satisfied
- note that in the VEC with consumption and income, the estimated adjustment parameter $\gamma_2 = -0.046$ is negative so is not consistent with stable relationship
- note also that it is not statistically significant
- we thus proceed to test a restriction $\gamma_2 = 0$ and reestimate the VEC model with this restriction

Application 1: Consumption Expenditures and Disposable Income

- to estimate a VEC model in EViews with restriction $\gamma_2 = 0$, in the estimation dialog window under "VEC Restrictions" tab enter `B(1,1)=1, A(2,1)=0`
- as shown below, the chi-square test statistic for the hypothesis $H_0 : \gamma_2 = 0$ is 1.393, its associated p-value is 0.2878, so we do not reject this hypothesis

Vector Error Correction Estimates
Date: 04/05/18 Time: 19:18
Sample: 1955Q1 2010Q4
Included observations: 224
Standard errors in ( ) & t-statistics in [ ]

Cointegration Restrictions:
B(1,1)=1, A(2,1)=0
Convergence achieved after 2 iterations.
Restrictions identify all cointegrating vectors
LR test for binding restrictions (rank = 1):

Chi-square(1)	1.393291
Probability	0.237850

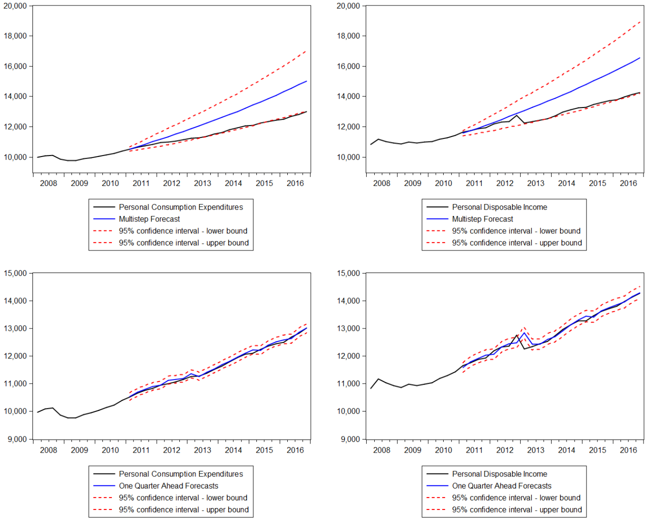
Cointegrating Eq:	CointEq1
LOG(NPCE(-1))	1.000000
LOG(NPDI(-1))	-1.012807
	(0.00266)
	[-380.816]
C	0.222564

Error Correction:	D(LOG(NPCE))	D(LOG(NPDI))
CointEq1	-0.128277	0.000000
	(0.02775)	(0.00000)
	[-4.62234]	[NA]

Application 1: Consumption Expenditures and Disposable Income

- to create a forecast using an estimated VEC model in EViews click on **Forecast** button or choose **Proc** → **Forecast...**
- the window that opens is the same as the one for Vector Autoregressive Models
- EViews will create a forecast for all variables in th VEC model, and by default store them in time series with suffix '_f'
- to create multistep forecasts set "Method" to "Dynamic forecast"
- to create a sequence of 1-step ahead forecasts set "Method" to "Static forecast"

Application 1: Consumption Expenditures and Disposable Income



Application 2: Pairs Trading

- cointegration and error correction model are used in the **pairs trading** strategy
- arbitrage pricing theory - if two stocks have similar characteristics, their prices must be more or less the same
- pairs trading involves selling the higher priced stock and buying the lower priced stock with the hope that the mispricing will correct itself in the future
- this strategy has been used on Wall Street for more than twenty years

Application 2: Pairs Trading

- consider two stocks with log prices $p_{i,t} = \log P_{i,t}$ for $i = 1, 2$ that follow random walk $p_{i,t} = p_{i,t-1} + r_{i,t}$ where $r_{i,t}$ are the serially uncorrelated log returns
- if the two stocks have similar risk factors, $p_{1,t}$ and $p_{2,t}$ will be driven by a common stochastic trend and cointegrated
- linear combination $z_t = p_{1,t} - \beta p_{2,t}$ will thus be $I(0)$ for some parameter β
- the stationary series z_t is referred to as the spread between the two log stock prices
- the two price series will follow error correction model

$$\begin{bmatrix} \Delta p_{1,t} \\ \Delta p_{2,t} \end{bmatrix} = \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} \begin{bmatrix} p_{1,t-1} - \beta p_{2,t-1} - \mu \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}$$

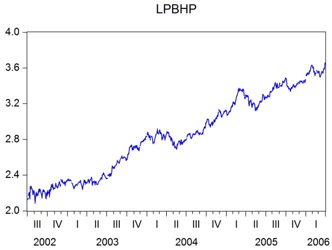
- reversion to the equilibrium requires $\gamma_1 \leq 0$ and $\gamma_2 \geq 0$

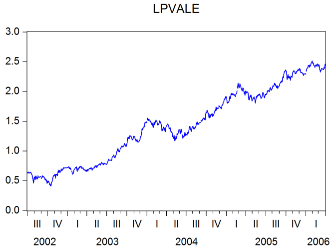
Application 2: Pairs Trading

- since spread z_t is $I(0)$ it is mean reverting
- trade are carried out when $z_t = p_{1,t} - \beta p_{2,t}$ deviates substantially from its mean μ
- one possible trading strategy
  - buy a share of stock 1 and short β shares of stock 2 at time t if $z_t = \mu - s$
  - unwind the position at time $t + i$ if $w_{t+i} = \mu + s$
- here s is chosen such that $2s > \eta$, where η is the costs of carrying out the two trades
- net profit is $2s - \eta$
- a modified trading strategy: if $s > \eta$ it is possible to unwind the position at time $t + i'$ if $w_{t+i'} = \mu$ which shortens the holding period of the portfolio

Application 2: Pairs Trading

- stock price data on two multinational companies, Billiton Ltd. (BHP) and Vale S.A. (VALE), that belong to natural resources industry and face similar risk factors





Application 2: Pairs Trading

- including 1 lag in cointegration test is suggested by SC, 2 lags are suggested by AIC

VAR Lag Order Selection Criteria
Endogenous variables: LPBHP LPVALE
Exogenous variables: C
Date: 04/20/17 Time: 04:27
Sample: 7/01/2002 3/31/2006
Included observations: 941

Lag	LogL	LR	FPE	AIC	SC	HQ
0	752.1508	NA	0.000696	-1.594369	-1.584068	-1.590442
1	4739.160	7948.596	1.47e-07	-10.05985	-10.02895*	-10.04807
2	4751.479	24.50836	1.44e-07*	-10.07753*	-10.02602	-10.05790*
3	4753.872	4.749374	1.45e-07	-10.07412	-10.00200	-10.04663
4	4755.723	3.666883	1.45e-07	-10.06955	-9.976834	-10.03421
5	4760.663	9.764458*	1.45e-07	-10.07155	-9.958229	-10.02835

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Application 2: Pairs Trading

- the two log price $p_{1,t}$ and $p_{2,t}$ are cointegrated

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.040602	39.13768	15.49471	0.0000
At most 1	1.02E-05	0.009583	3.841466	0.9217

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.040602	39.12810	14.26460	0.0000
At most 1	1.02E-05	0.009583	3.841466	0.9217

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Application 2: Pairs Trading

- estimated VEC is stable, $\gamma_1 = -0.062721 < 0$ and $\gamma_2 = 0.033030 > 0$

LPBHP(-1)	1.000000	
LPVALE(-1)	-0.717784 (0.01118) [-64.1918]	
C	-1.821138	
Error Correction:	D(LPBHP)	D(LPVALE)
CointEq1	-0.062721 (0.01461) [-4.29201]	0.033030 (0.01692) [1.95231]
D(LPBHP(-1))	-0.114859 (0.03671) [-3.12879]	0.052833 (0.04250) [1.24311]
D(LPVALE(-1))	0.069178 (0.03205) [2.15863]	0.045228 (0.03710) [1.21902]
C	0.001667 (0.00063) [2.64118]	0.001766 (0.00073) [2.41816]

Application 2: Pairs Trading

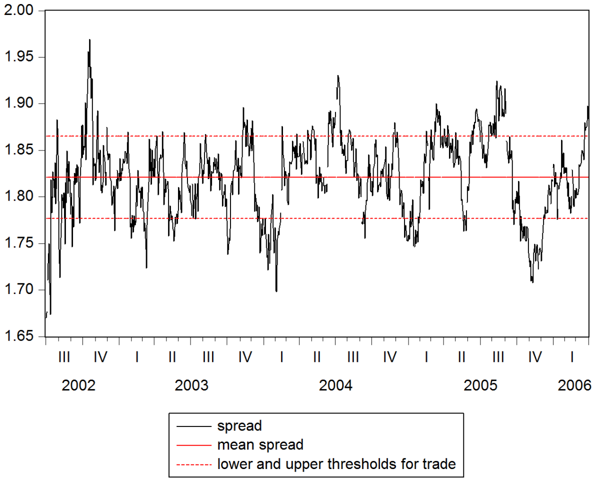
- estimated VEC model takes form

$$\begin{bmatrix}\Delta p_{1,t}\\ \Delta p_{2,t}\end{bmatrix} = \begin{bmatrix}0.001\\ 0.001\end{bmatrix} + \begin{bmatrix}-0.062\\ 0.033\end{bmatrix}[p_{1,t-1} - 0.717 p_{2,t-1} - 1.821] + \begin{bmatrix}-0.11 & 0.06\\ 0.05 & 0.04\end{bmatrix}\begin{bmatrix}\Delta p_{1,t-1}\\ \Delta p_{2,t-1}\end{bmatrix} + \begin{bmatrix}\varepsilon_{1,t}\\ \varepsilon_{2,t}\end{bmatrix}$$

Application 2: Pairs Trading

- the spread is thus calculated as $z_t = p_{1,t} - \hat{\beta} p_{2,t} = p_{1,t} - 0.717 p_{2,t}$
- the mean spread is 1.821
- the standard deviation is 0.044
- given that $\hat{\sigma}$ is quite large, it is possible to choose trading strategy by setting $s = 0.045$ which yields log return for each pairs trading $2s = 0.09$
- as shown in the figure on the next slide, z_t moves between $\hat{\mu} - 0.045$ and $\hat{\mu} + 0.045$ relatively often, so there are many pairs-trading opportunities

Application 2: Pairs Trading



Application 2: Pairs Trading

- note that this illustrative example is based on in-sample analysis
- a realistic demonstration would require to assess the out-of-sample performance
- identifying cointegrated pairs of stocks that share similar risk factors may by quite challenging
- main issue: if a lot of traders exploit a particular pairs trading strategy, the two stocks may cease to be cointegrated

Application 3: Money Demand

- consider the money demand equation from Intermediate Macroeconomics

$$M^d = PL(Y, i)$$

  where M^d is the demand for money, P is the price level, Y real income, i nominal interest rate on bonds

- this theory predicts that M^d in increasing when P increases, Y increases, or i decreases

Application 3: Money Demand

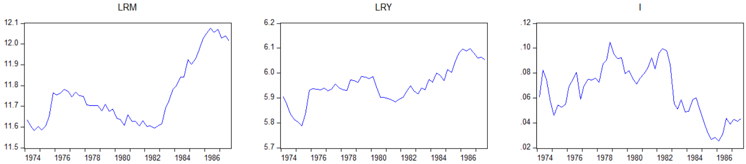
- data for Denmark, from Johansen's study which invented the cointegration tests, for the period 1974Q1-1987Q3

$$\boldsymbol{y}_t = (\log(M2_t/P_t), \log Y_t, i_t)'$$

  where $\log(M2_t/P_t)$ is log of money stock M2 deflated by price index, $\log Y_t$ is log of real income, i_t is the spread between bond rate and deposit rate

- based on unit root tests all series are confirmed to be $I(1)$

Application 3: Money Demand



Application 3: Money Demand

- based on the information criteria either 1 or 2 lags should be considered in the cointegration analysis

VAR Lag Order Selection Criteria
Endogenous variables: LRM LRY I
Exogenous variables: C
Date: 04/20/17 Time: 05:39
Sample: 1974Q1 1987Q3
Included observations: 50

Lag	LogL	LR	FPE	AIC	SC	HQ
0	275.7640	NA	3.67e-09	-10.91056	-10.79584	-10.86687
1	405.3392	238.4184	2.95e-11	-15.73357	-15.27468*	-15.55882*
2	417.3081	20.58647*	2.63e-11*	-15.85232*	-15.04927	-15.54652
3	422.5378	8.367631	3.09e-11	-15.70151	-14.55430	-15.26465
4	429.1015	9.714269	3.48e-11	-15.60406	-14.11268	-15.03614
5	438.5726	12.88061	3.53e-11	-15.62290	-13.78736	-14.92392

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Application 3: Money Demand

- data suggests using either Case 3 cointegration test or Case 2 cointegration test
- regardless of the choice of Case 2 or Case 3 test, or using 1 or 2 lags
  - H_0 of no cointegration is rejected by both trace and maximum eigenvalue tests
  - H_1 of 1 cointegration relationship can not be rejected
  - estimated coefficient in the cointegration relationship do not change much

Application 3: Money Demand

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.420763	37.84590	29.79707	0.0048
At most 1	0.138097	9.451659	15.49471	0.3252
At most 2	0.032607	1.723837	3.841466	0.1892

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.420763	28.39424	21.13162	0.0040
At most 1	0.138097	7.727822	14.26460	0.4070
At most 2	0.032607	1.723837	3.841466	0.1892

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Application 3: Money Demand

- comparing the AIC and SC for VEC models with 1 and 2 lags and based on Case 2 and Case 3 cointegration shows that even though the differences are small, Case 2 with 1 lag is preferred

	AIC	SC
Case 2 with 1 lag	-15.745	-15.150
Case 2 with 2 lags	-15.607	-14.669
Case 3 with 1 lag	-15.698	-15.028
Case 3 with 2 lags	-15.550	-14.536

Application 3: Money Demand

- note: in the VEC model β_2 is close to -1, γ_2 and γ_3 have wrong signs inconsistent with stable long run self correcting relationship, but both are not significant

LRM(-1)	1.000000		
LRY(-1)	-0.929265		
	(0.15158)		
	[-6.13049]		
I(-1)	6.218156		
	(0.56117)		
	[11.0807]		
C	-6.655966		
	(0.92109)		
	[-7.22621]		
Error Correction:	D(LRM)	D(LRY)	D(I)
CointEq1	-0.283193	-0.002221	-0.018758
	(0.06164)	(0.05118)	(0.01949)
	[-4.59417]	[-0.04339]	[-0.96226]
D(LRM(-1))	-0.142296	0.318248	-0.021808
	(0.14326)	(0.11894)	(0.04531)
	[-0.99326]	[2.67580]	[-0.48135]
D(LRY(-1))	0.132728	-0.027667	0.119406
	(0.18122)	(0.15045)	(0.05731)
	[0.73241]	[-0.18389]	[2.08348]
D(I(-1))	0.650949	0.171624	0.092720
	(0.47435)	(0.39381)	(0.15001)
	[1.37230]	[0.43581]	[0.61809]
R-squared	0.283992	0.151964	0.110557
Adj. R-squared	0.240155	0.100043	0.056101
Sum sq. resids	0.040757	0.028091	0.004076
S.E. equation	0.028840	0.023943	0.009121
F-statistic	6.478340	2.926845	2.030215
Log likelihood	114.8124	124.6751	175.8280
Akaike AIC	-4.181602	-4.553778	-6.484077
Schwarz SC	-4.032900	-4.405077	-6.335376
Mean dependent	0.007757	0.003340	-0.000730
S.D. dependent	0.033086	0.025239	0.009388

Application 3: Money Demand

- we impose restrictions $\beta_1 = 1, \beta_2 = -1$ by entering `B(1,1)=1, B(1,2)=-1`
- test statistic is 0.137, p-value 0.710 so we can not reject this hypothesis

Cointegration Restrictions:
B(1,1)=1,B(1,2)=-1
Convergence achieved after 3 iterations.
Restrictions identify all cointegrating vectors
LR test for binding restrictions (rank = 1):

Chi-square(1)	0.137866
Probability	0.710412

Cointegrating Eq:	CointEq1
LRM(-1)	1.000000
LRY(-1)	-1.000000
I(-1)	5.984651
	(0.46908)
	[12.7583]
C	-6.218335
	(0.03338)
	[-186.308]

Error Correction:	D(LRM)	D(LRY)	D(I)
CointEq1	-0.292609	0.005149	-0.018778
	(0.06522)	(0.05376)	(0.02050)
	[-4.48644]	[0.09577]	[-0.91604]
D(LRM(-1))	-0.128205	0.322280	-0.020556
	(0.14365)	(0.11841)	(0.04515)
	[-0.89250]	[2.72167]	[-0.45530]
D(LRY(-1))	0.116634	-0.026915	0.118411
	(0.18262)	(0.15054)	(0.05740)
	[0.63868]	[-0.17880]	[2.06299]
D(I(-1))	0.651610	0.144809	0.090587
	(0.47966)	(0.39540)	(0.15076)
	[1.35849]	[0.36624]	[0.60087]

Application 3: Money Demand

- we can impose and test restrictions $\beta_1 = 1, \beta_2 = -1$ and in addition $\gamma_2 = \gamma_3 = 0$ by entering `B(1,1)=1, B(1,2)=-1, A(2,1)=0, A(3,1)=0`
- test statistic is 1.025, p-value 0.795 so we can not reject this hypothesis

Cointegration Restrictions:
B(1,1)=1,B(1,2)=-1,A(2,1)=0,A(3,1)=0
Convergence achieved after 6 iterations.
Restrictions identify all cointegrating vectors
LR test for binding restrictions (rank = 1):

Chi-square(3)	1.025677
Probability	0.795039

Cointegrating Eq:	CointEq1
LRM(-1)	1.000000
LRY(-1)	-1.000000
I(-1)	5.915743 (0.46977) [12.5927]
C	-6.214915 (0.03343) [-185.930]

Error Correction:	D(LRM)	D(LRY)	D(I)
CointEq1	-0.319640 (0.04666) [-6.85047]	0.000000 (0.00000) [NA]	0.000000 (0.00000) [NA]
D(LRM(-1))	-0.127397 (0.14336) [-0.88865]	0.322236 (0.11835) [2.72266]	-0.020037 (0.04516) [-0.44368]
D(LRY(-1))	0.115727 (0.18235) [0.63464]	-0.026906 (0.15054) [-0.17873]	0.118460 (0.05744) [2.06225]
D(I(-1))	0.654933 (0.47888) [1.36762]	0.144957 (0.39535) [0.36666]	0.087574 (0.15085) [0.58053]

Summary VAR vs VEC

- if variables $\boldsymbol{y}_t$ are $I(0)$ we don't difference data and estimate VAR in levels
  - if they grow along a deterministic trend, this trend is included in the VAR
- if variables $\boldsymbol{y}_t$ are $I(1)$ we first test them for cointegration
  - if they are cointegrated we estimate a VEC model
  - if they are not cointegrated we difference the data and estimate a VAR model on first differences Δy_t