Eco 4306 Economic and Business Forecasting

Chapter 9: Assessment of Forecasts and Combination of Forecasts

Motivation

- we discussed how to asses the models based on **in-sample evaluation**
  - parameters of the model are statistically significant
  - residuals should be white noise (no significant lags in correlograms for residuals)
  - AIC and SIC should be low
- we will now focus on **out-of-sample evaluation**
  - assess the forecasting ability of each model

9.1 Optimal Forecast

- uncertainty is inherent in any forecast, forecaster will necessarily make forecast errors which are costly
- forecaster would like to minimize the expected costs associated with the forecast errors
- this leads to the concept optimal forecast associated with forecasters loss function
- we should thus also evaluate forecasts from competing models using forecaster's loss function
- we will see how to combine forecasts from several models to achieve lower loss than using a forecast from a single model

9.1 Optimal Forecast

- suppose that the forecaster has a loss function

$$L(e_{t,h}) = L(y_{t+h} - f_{t,h})$$

  where $f_{t,h}$ is the forecast at time t of the random variable Y_{t+h}, which has a conditional probability density function $f(y_{t+h}|I_t)$
- the expected loss associated with potential forecast errors is

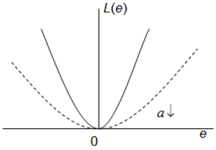
$$E(L(y_{t+h} - f_{t,h})) = \int L(y_{t+h} - f_{t,h}) f(y_{t+h}|I_t) dy_{t+h}$$

- **optimal forecast is the forecast that minimizes the expected loss**

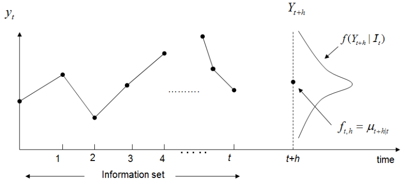
9.1.1 Symmetric and Asymmetric Loss Functions

under symmetric quadratic loss functions $L(e_{t,h}) = ae_{t,h}^2$

- positive or negative errors of the same magnitude have identical costs



- the optimal forecast is the conditional mean $f_{t,h}^* = \mu_{t+h|t}$



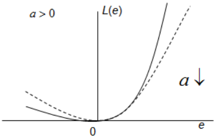
- forecast is unbiased, mean forecast error is zero

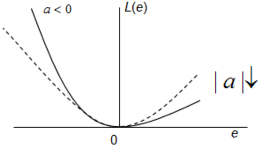
$$E(e_{t,h}) = E(Y_{t+h} - \mu_{t+h|t}) = \mu_{t+h|t} - \mu_{t+h|t} = 0$$

9.1.1 Symmetric and Asymmetric Loss Functions

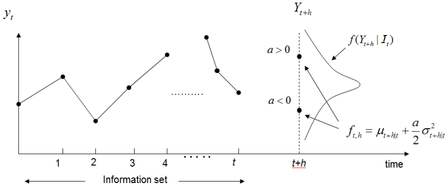
under asymmetric linex loss function $L(e_{t,h}) = exp(ae_{t,h}) - ae_{t,h} - 1$

- positive and negative errors of the same magnitude have different costs





- if Y_{t+h} is normally distributed, the optimal forecast is $f^*_{t,h} = \mu_{t+h|t} + \frac{a}{2}\sigma^2_{t+h|t}$
- if $a < 0$ forecaster wants to avoid negative errors, so the forecast is pushed down to make $y_{t+h} > f_{t,h}$ more likely; optimal forecast will be smaller than the conditional mean and is thus biased, $E(e_{t,h}) > 0$
- opposite applies when $a > 0$: forecaster wants to avoid positive forecast errors, forecast will be pushed above the conditional mean and is thus biased $E(e_{t,h}) < 0$



9.1.1 Symmetric and Asymmetric Loss Functions

- consider again the case of forecasting quarterly House Price Index in the San Diego MSA from lec10slides.pdf
- when a mortgage bank is forecasting home prices, negative forecast errors could be more damaging than positive forecast errors: when house prices go down, home equity goes down and homeowners with large mortgages may find that the value of the house is less than the amount of the mortgage, creating incentives to default
- thus the bank will prefer to act conservatively, and bias its forecast downward
- assume that the forecasters at the bank choose a Linex loss function with $a = -\frac{1}{2}$, the optimal forecast is then constructed using

$$f^*_{t,h} = \hat{\mu}_{t+h|t} - \frac{1}{4}\hat{\sigma}^2_{t+h|t}$$

9.1.2 Testing the Optimality of the Forecast

- to perform out of sample evaluation of model forecasts we start by spliting the sample into **prediction sample** and **estimation sample**
- in practice, prediction sample should be no more than 10% of the sample

9.2 Assessment of Forecasts

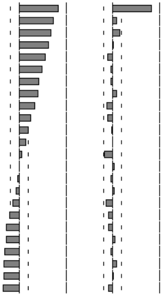
- suppose that we have constructed the sequence of h step ahead forecasts using either fixed, recursive, or rolling scheme as discussed in lec06slides.pdf
- we can use the out-of-sample forecast errors $\{e_{t,h}, e_{t+1,h}, e_{t+2,h}, \ldots, e_{T-h,h}\}$ to calculate the **sample mean loss** $\bar{L}$
- ranking the forecasts from different model by their sample average loss is a simple and quick procedure to choose the best forecast

9.2 Assessment of Forecasts

- consider the house price index for California 1975Q1-2007Q4, variable **ghpca** in ghpca.wf1
- suppose that we want to build a model with estimation sample 1975Q1-2002Q4, and leave 2003Q1-2007Q4 as prediction sample for forecast evaluation
- the correlogram for 1975Q1-2002Q4 shows that ACF declines gradually and PACF has significant lag 1 and marginally significant lag 3

Date: 04/03/17 Time: 20:10
Sample: 1975Q1 2002Q4
Included observations: 111

Autocorrelation | Partial Correlation



	AC	PAC	Q-Stat	Prob
1	0.855	0.855	83.377	0.000
2	0.755	0.090	149.02	0.000
3	0.707	0.161	207.15	0.000
4	0.654	0.012	257.31	0.000
5	0.571	-0.105	295.90	0.000
6	0.499	-0.033	325.65	0.000
7	0.439	-0.028	348.89	0.000
8	0.408	0.088	369.18	0.000
9	0.336	-0.129	383.09	0.000
10	0.249	-0.115	390.78	0.000
11	0.191	-0.018	395.33	0.000
12	0.151	0.008	398.23	0.000
13	0.058	-0.186	398.66	0.000
14	-0.004	0.028	398.66	0.000
15	-0.049	-0.028	398.98	0.000
16	-0.078	0.026	399.78	0.000
17	-0.150	-0.156	402.80	0.000
18	-0.221	-0.082	409.40	0.000
19	-0.281	-0.090	420.14	0.000
20	-0.293	0.063	432.01	0.000
21	-0.335	-0.049	447.64	0.000
22	-0.352	0.086	465.09	0.000
23	-0.348	0.022	482.35	0.000
24	-0.357	-0.097	500.68	0.000

9.2 Assessment of Forecasts

- we thus estimate an AR(3) model

$$y_t = c + \phi_1 y_{t-1} + \phi_3 y_{t-3} + \varepsilon_t$$

Dependent Variable: GHPCA
Method: ARMA Maximum Likelihood (BFGS)
Date: 04/03/17 Time: 20:05
Sample: 1975Q2 2002Q4
Included observations: 111
Convergence achieved after 8 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.211330	0.927358	2.384549	0.0189
AR(1)	0.733166	0.068178	10.75367	0.0000
AR(3)	0.155077	0.074522	2.080945	0.0398
SIGMASQ	1.098276	0.124433	8.826219	0.0000

R-squared	0.744947	Mean dependent var	2.016012
Adjusted R-squared	0.737796	S.D. dependent var	2.084519
S.E. of regression	1.067396	Akaike info criterion	3.016160
Sum squared resid	121.9087	Schwarz criterion	3.113801
Log likelihood	-163.3969	Hannan-Quinn criter.	3.055770
F-statistic	104.1737	Durbin-Watson stat	1.959416
Prob(F-statistic)	0.000000		

Inverted AR Roots	.92	-.09+.40i	-.09-.40i

9.2 Assessment of Forecasts

- we will now use the estimated AR(3) to obtain one step ahead fixed scheme forecasts $f_{t,1}, f_{t+1,1}, f_{t+2,1}, \ldots, f_{T-1,1}$ for the prediction sample 2003Q1-2007Q4
- to do it in EViews click on **Proc** $\rightarrow$ **Forecast** and choose "Static forecast" for "Method"" option, enter **ghpca_f_ar3_fix** in the "Forecast name", and **2003Q1-2007Q4** in "Forecast sample"
- next, construct the forecast errors by subtracting the forecast from the actual series: $e_{t+j,1} = y_{t+j+1} - f_{t+j,1}$ for $j = 0, 1, \ldots, T-t-1$
- to do this in EViews choose **Object** $\rightarrow$ **Generate series** and enter **ghpca_e_ar3_fix = ghpca - ghpca_f_ar3_fix**

9.2 Assessment of Forecasts

- we will compare the forecast from the AR(3) model for the 2003Q1-2007Q4 period with two alternative forecasts
  1. a naive forecast, constructed by choosing **Object** → **Generate series** and entering **ghpca_f_naive = ghpca(-1)**
  2. a four quarter simple moving average forecast, obtained by **Object** → **Generate series** and entering **ghpca_f_ma = 1/4*(ghpca(-1)+ghpca(-2)+ghpca(-3)+ghpca(-4))**
- for these alternative forecasts we calculate the forecast errors
  1. for naive forecast we generate **ghpca_e_naive = ghpca - ghpca_f_naive**
  2. for simple moving average forecast we generate **ghpca_e_ma = ghpca - ghpca_f_ma**

precision of a forecast should be assessed based on a measure consistent with the choice of the loss function

Mean Squared Error

$$MSE = \frac{1}{T-t-h+1} \sum_{j=0}^{T-h-t} e_{t+j,h}^2$$

is the sample average loss corresponding to a symmetric quadratic loss function; it is customary to report the root mean squared error $RMSE = \sqrt{MSE}$

Mean Absolute Error

$$MAE = \frac{1}{T-t-h+1} \sum_{j=0}^{T-h-t} |e_{t+j,h}|$$

is the sample average loss corresponding to a symmetric absolute value loss function precision of a forecast should be assessed based on a measure consistent with the choice of the loss function

Mean Absolute Percentage Error

$$MAPE = \frac{1}{T-t-h+1}\sum_{j=0}^{T-h-t}\left|\frac{e_{t+j,h}}{y_{t+j+h}}\right|$$

is the sample average loss corresponding to a loss function $L(e,y) = \frac{|e|}{y}$

Mean Loss

$$\bar{L} = \frac{1}{T-t-h+1}\sum_{j=0}^{T-l-t} L(e_{t+j,h})$$

for example with linex loss function

$$\bar{L} = \frac{1}{T-t-h+1}\sum_{j=0}^{T-l-t}\Big(\exp(ae_{t+j,h}) - ae_{t+j,h} - 1\Big)$$

9.2 Assessment of Forecasts

- to obtain MSE for the forecasts, we first need to generate the series for the squared forecast errors (these also measure the loss in each period given symmetric quadratic loss function)
- for the one quarter ahead fixed scheme forecast from the AR(3) model choose **Object** $\rightarrow$ **Generate Series** and enter **ghpca_l_ar3_fix = ghpca_e_ar3_fix^2**
- series **ghpca_l_naive = ghpca_e_naive^2** and **ghpca_l_ma = ghpca_e_ma^2** are constructed in EViews in a similar way

9.2 Assessment of Forecasts

- then, to obtain MSE for these forecasts select all three series **ghpca_l_ar3_fix**, **ghpca_l_naive**, **ghpca_l_ma**, as group and choose **View** → **Descriptive Stats** → **Common Sample**
- the numbers in the first row which show the mean of the squared errors are the MSEs for the three forecasts

Date: 03/05/18 Time: 14:20
Sample: 2003Q1 2007Q4

	GHPCA_L_AR3_FIX	GHPCA_L_NAIVE	GHPCA_L_MA
Mean	3.82086	4.399913	4.981485
Median	1.424206	1.216233	3.45836
Maximum	20.84603	37.51581	27.93979
Minimum	0.016053	0.008369	0.003614
Std. Dev.	5.817161	8.755445	6.311292
Skewness	2.090813	3.039272	2.516509
Kurtosis	6.219916	11.62986	9.868731
Jarque-Bera	23.21155	92.85265	60.4256
Probability	0.000009	0	0
Sum	76.4172	87.99826	99.62971
Sum Sq. Dev.	642.948	1456.499	756.8156
Observations	20	20	20

9.2.2 Statistical Evaluation of the Average Loss

- the comparison of the three forecasts shows that in this case AR(3) is most precise since its MSE is the lowest, and the simple moving average is least precise since its MSE is the highest
- in general, when we have several competing forecasts, the preferred one is the forecast that has the lowest average loss, based on either MSE, MAE, MAPE or some general mean loss
- however, since in practice we work with sample information, so we need to consider sampling variation in our assessment of forecasts
- this means that we need to test whether the calculated difference in sample mean loss of any two competing forecasts is statistically significant or not
- in other words, it is possible that in our example the 3.8208 MSE of the forecast from the AR(3) model is not statistically lower than the 4.3999 MSE from the naive forecast

9.2.2.1 Test of Equal Predictive Ability

- **hypothesis of equal predictive ability** of forecasts from two models A and B is written in terms of the unconditional expectation of the loss difference

$$H_0 : E(L(e_{t,h}^{(A)})) - E(L(e_{t,h}^{(B)})) = E(\Delta L_{t,h}) = 0$$

- the test of this hypothesis can be easily carried out by estimaing a simple regression model

$$\Delta L_{t+j,h} = \beta_0 + \varepsilon_{t+j} \qquad \text{for } j = 0, 1, 2, \ldots, T - t - h$$

- if both forecasts deliver the same expected loss, coefficient β is zero
- this means that the hypothesis of equal predictive ability of two forecasts A and B can be restated as

$$H_0 : \beta_0 = 0$$

- consequently, if β_0 is statistically significant we reject the hypothesis of equal predictive ability of forecasts A and B

9.2.2.1 Test of Equal Predictive Ability

- to perform the test of equal predictive ability that compares the naive and fixed scheme forecast generated by the AR(3) model we need to generate series for loss differential
- we do so by choosing **Object** $\rightarrow$ **Generate Series** and entering
  dl_naive = ghpca_l_naive - ghpca_l_ar3_fix
- then we estimate the OLS model

$$\Delta L_{t+j,h} = \beta_0 + \varepsilon_{t+j} \qquad \text{for } j = 0, 1, 2, \ldots, T - t - h$$

by choosing **Object** $\rightarrow$ **New Object** $\rightarrow$ **Equation** and entering
dl_naive c in the specification dialog

9.2.2.1 Test of Equal Predictive Ability

- the results of the estimation below show that β_0 is not statistically significant - its p-value is 0.5263 so larger than 0.1
- thus in this case we can not reject the null hypothesis of the equal predictive ability of AR(3) model and the naive forecast

Dependent Variable: DL_NAIVE
Method: Least Squares
Date: 04/03/17 Time: 20:05
Sample: 2003Q1 2007Q4
Included observations: 20
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 3.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.579053	0.897018	0.645531	0.5263

R-squared	0.000000	Mean dependent var	0.579053
Adjusted R-squared	0.000000	S.D. dependent var	4.642825
S.E. of regression	4.642825	Akaike info criterion	5.957230
Sum squared resid	409.5606	Schwarz criterion	6.007016
Log likelihood	-58.57230	Hannan-Quinn criter.	5.966949
Durbin-Watson stat	2.342480		

9.3 Combination of Forecasts

- often we may not able to find a unique model that produces best forecast all the time
  - some models may work better in high volatility times than in calmer times
  - some models may adjust faster than others to regime changes caused by new policies
  - best model can change over the time span of the series
  - different models and forecasts may be based on different information sets
- a combination of forecasts can thus often do better than any individual forecast

9.3.1 Simple Linear Combinations

- suppose that we have n forecasts $\{f_{t,h}^{(1)}, f_{t,h}^{(2)}, \ldots, f_{t,h}^{(n)}\}$ with their corresponding forecast errors $\{e_{t,h}^{(1)}, e_{t,h}^{(2)}, \ldots, e_{t,h}^{(n)}\}$
- a linear combination of forecasts is given by

$$f_{t,h}^{c} = \omega_1 f_{t,h}^{(1)} + \omega_2 f_{t,h}^{(2)} + \ldots + \omega_n f_{t,h}^{(n)}$$

  where ω_i is the weight assigned to forecast $f_{t,h}^{(i)}$
- note: weights do not need to add to 1 or be strictly positive
- example 1: when the weights are $\omega_i = 1/n$, we have an equal-weighted forecast that is the arithmetic average of the individual forecasts
- example 2: forecaster may wish to weight forecasts so that those with a lower MSE are assigned a larger weight than those with larger MSE by setting $\omega_i = \frac{1/MSE_i}{\sum_{j=1}^{n} 1/MSE_j}$ where MSE_i is the mean squared error of forecast i

9.3.2 Optimal Linear Combinations

- example 3: it is also possible to estimate the weights by regressing the realized values on the individual forecasts

$$y_{t+h} = \omega_0 + \omega_1 f_{t,h}^{(1)} + \omega_2 f_{t,h}^{(2)} + \ldots + \omega_n f_{t,h}^{(n)} + \varepsilon_{t+h}$$

- using OLS we can obtain estimates for ω_i, which may be positive or negative and may not total 1
- if the individual forecasts are unbiased, the constant in the regression will be zero, i.e., $\omega_0 = 0$
- if the the forecasts are not unbiased, the constant will pick up the bias assuming that this is not time-varying