Eco 4306 Economic and Business Forecasting

Chapter 7: Seasonal Cycles

Motivation

- production, consumption, and other economic activities are generally organized according to the calendar (quarters, months, days, hours, and special holidays)
- these actions appear in the data as a seasonal cycle at the quarterly, monthly, daily, or hourly frequency
- examples
  - retail sales: high in November and December
  - travel industry: people travel more in the summer, number of passengers traveling by air, train, car, and boat substantially increases, expenditures for gas are highest in summer
  - construction: start of residential units occurs in the beginning of spring
  - food industry: sales of liquor and alcohol tend to increase in the winter months, sales of ice cream in the summer months
  - entertainment industry: sales of tickets are higher on weekends than on weekdays
  - stock market: the volume of trading is larger at the beginning and at the end of the trading day
- **seasonal cycle**: periodic fluctuation in the data associated with the calendar

Motivation

- in many economic databases, we find seasonally adjusted time series for which the seasonal cycle has been removed
- macrolevel: policy makers, institutions, and economic forecasters in general are more concerned with the analysis of trends
- microlevel: businesses generally are very much interested in forecasting sales every month or every quarter; thus, they need the joint analysis of the seasonal and nonseasonal components in sales

7.3.1 Deterministic and Stochastic Seasonal Cycles

- we will distinguish **deterministic seasonality** and **stochastic seasonality**
- deterministic seasonality: captured in a regression model by assigning specific *constant* effects to each month or quarter
- stochastic seasonality: MA and AR specifications have natural extensions to model the seasonal component of a series, size of the seasonal effect is no longer constant

7.3.1 Deterministic and Stochastic Seasonal Cycles

deterministic seasonality

- suppose that we collect a quarterly time series $\{y_t\}$, e.g. retail sales, and wish to analyze the seasonal component
- construct four time series dummy variables $Q1, Q2, Q3, Q4$ so that Qi will assign a value 1 to the quarter i and 0 otherwise

obs	SALES ($)	Q1	Q2	Q3	Q4
1999Q4	768726.0	0.000000	0.000000	0.000000	1.000000
2000Q1	696048.0	1.000000	0.000000	0.000000	0.000000
2000Q2	753211.0	0.000000	1.000000	0.000000	0.000000
2000Q3	746875.0	0.000000	0.000000	1.000000	0.000000
2000Q4	792622.0	0.000000	0.000000	0.000000	1.000000
2001Q1	704757.0	1.000000	0.000000	0.000000	0.000000
2001Q2	779011.0	0.000000	1.000000	0.000000	0.000000
2001Q3	756128.0	0.000000	0.000000	1.000000	0.000000
2001Q4	827829.0	0.000000	0.000000	0.000000	1.000000
2002Q1	717302.0	1.000000	0.000000	0.000000	0.000000
2002Q2	790486.0	0.000000	1.000000	0.000000	0.000000
2002Q3	792657.0	0.000000	0.000000	1.000000	0.000000
2002Q4	833877.0	0.000000	0.000000	0.000000	1.000000
2003Q1	741233.0	1.000000	0.000000	0.000000	0.000000
2003Q2	819940.0	0.000000	1.000000	0.000000	0.000000

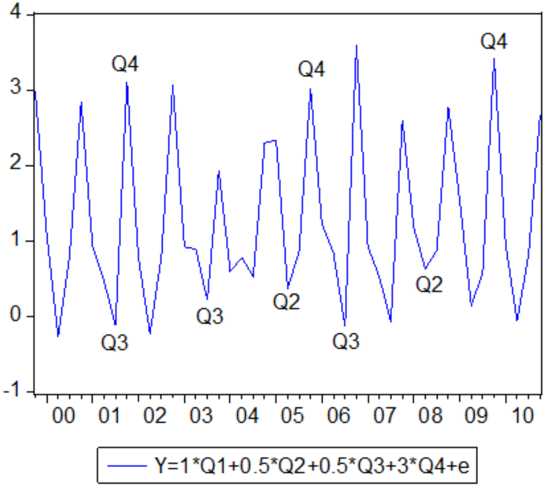
- estimate regression

$$Y_t = \beta_1 Q1_t + \beta_2 Q2_t + \beta_3 Q3_t + \beta_4 Q4_t + \varepsilon_t$$

- note that we are *not including constant* in the regression, that would lead to multicolinearity since $Q1_t + Q2_t + Q3_t + Q4_t = 1$
- β_i is interpreted as expected (average) sales in quarter i

7.3.1 Deterministic and Stochastic Seasonal Cycles

deterministic seasonality



7.3.1 Deterministic and Stochastic Seasonal Cycles

stochastic seasonality

- seasonal component is driven by random variables
- for example: consider quarterly seasonal AR(1) model

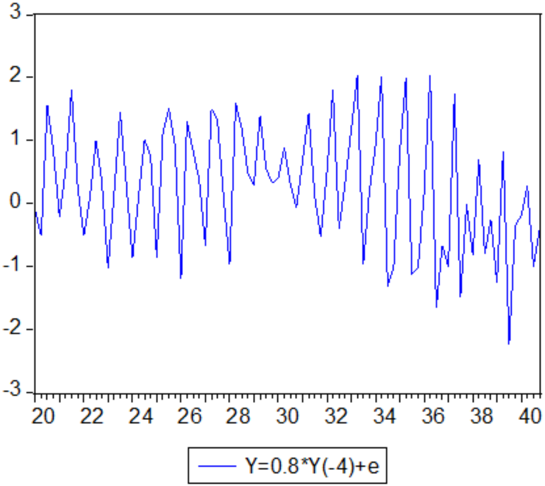
$$Y_t = c + \Phi Y_{t-4} + \varepsilon_t$$

  or equivalently using lag operator

$$(1 - \Phi L^4) Y_t = c + \varepsilon_t$$

7.3.1 Deterministic and Stochastic Seasonal Cycles

stochastic seasonality



7.3.2 Seasonal ARMA Models

- seasonal AR of order P, so an S-AR(P), is defined as

$$Y_t = c + \Phi_s Y_{t-s} + \Phi_{2s} Y_{t-2s} + \ldots + \Phi_{Ps} Y_{t-Ps} + \varepsilon_t$$

  where s refers to the frequency of the data

- using lag operator we can equivalently write S-AR(P) as

$$(1 - \Phi_s L^s - \Phi_{2s} L^{2s} - \ldots - \Phi_{Ps} L_{Ps}) Y_t = c + \varepsilon_t$$

- if we have quarterly data $s = 4$, for monthly data $s = 12$, for daily data, with five working days $s = 5$
- for example, an S-AR(1) for quarterly data is written as

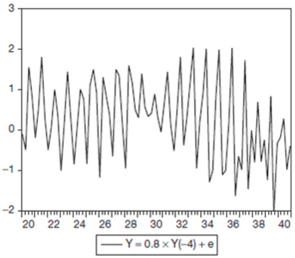
$$Y_t = c + \Phi_4 Y_{t-4} + \varepsilon_t$$

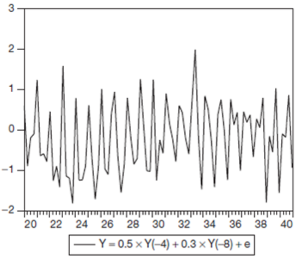
  and an S-AR(2) for monthly data as

$$Y_t = c + \Phi_{12} Y_{t-12} + \Phi_{24} Y_{t-24} + \varepsilon_t$$

7.3.2 Seasonal ARMA Models

- S-AR(1) and S-AR(2) for quarterly data and their AC and PAC functions





Included observations: 4201

Autocorrelation	Partial Correlation		AC	PAC
		1	-0.020	-0.020
		2	-0.027	-0.027
		3	-0.022	-0.023
		4	0.796	0.796
		5	-0.007	0.030
		6	-0.024	0.004
		7	-0.025	-0.009
		8	0.641	0.021
		9	0.012	0.027
		10	-0.017	0.014
		11	-0.014	0.036
		12	0.515	-0.001
		13	0.021	-0.005
		14	-0.011	0.004
		15	-0.001	0.015
		16	0.414	0.002
		17	0.025	-0.001
		18	-0.004	0.006
		19	0.004	-0.007
		20	0.337	0.012
		21	0.028	0.004

Included observations: 4197

Autocorrelation	Partial Correlation		AC	PAC
		1	-0.018	-0.018
		2	-0.109	-0.109
		3	-0.022	-0.026
		4	0.729	0.725
		5	-0.003	0.022
		6	-0.116	-0.033
		7	-0.032	-0.021
		8	0.668	0.290
		9	0.005	0.010
		10	-0.097	0.033
		11	-0.035	0.000
		12	0.560	0.005
		13	0.016	0.008
		14	-0.097	-0.005
		15	-0.033	0.014
		16	0.485	0.002
		17	0.021	0.003
		18	-0.088	-0.003
		19	-0.037	-0.006
		20	0.417	0.005
		21	0.009	-0.040

7.3.2 Seasonal ARMA Models

- stochastic seasonality can also be specified within MA models
- a seasonal MA of order Q, S-MA(q) is defined as

$$Y_t = \mu + \varepsilon_t + \Theta_s \varepsilon_{t-s} + \Theta_{2s} \varepsilon_{t-2s} + \ldots + \Theta_{Qs} \varepsilon_{t-Qs}$$

- using lag operator we can equivalently write S-AR(P) as

$$Y_t = \mu + (1 - \Theta_s L^s - \Theta_{2s} L^{2s} - \ldots - \Theta_{Qs} L_{Qs}) \varepsilon_t$$

- for example, an S-MA(1) for quarterly data is written as

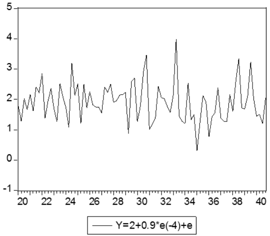
$$Y_t = \mu + \varepsilon_t + \Theta_4 \varepsilon_{t-4}$$

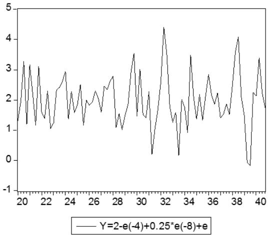
  and an S-MA(2) for monthly data as

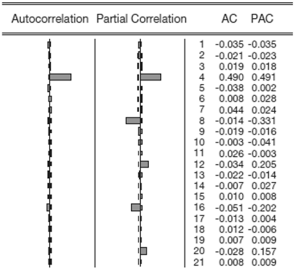
$$Y_t = \mu + \varepsilon_t + \Theta_{12} \varepsilon_{t-12} + \Theta_{24} \varepsilon_{t-24}$$

7.3.2 Seasonal ARMA Models

- S-MA(1) and S-MA(2) for quarterly data and their AC and PAC functions

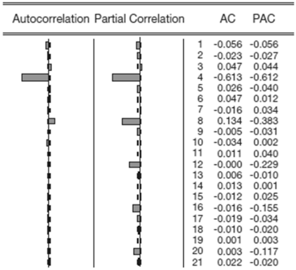






Included observations: 4193

Autocorrelation	Partial Correlation		AC	PAC
		1	-0.035	-0.035
		2	-0.021	-0.023
		3	0.019	0.018
		4	0.490	0.491
		5	-0.038	0.002
		6	0.008	0.028
		7	0.044	0.024
		8	-0.014	-0.331
		9	-0.019	-0.016
		10	-0.003	-0.041
		11	0.026	-0.003
		12	-0.034	0.205
		13	-0.022	-0.014
		14	-0.007	0.027
		15	0.010	0.008
		16	-0.051	-0.202
		17	-0.013	0.004
		18	0.012	-0.006
		19	0.007	0.009
		20	-0.028	0.157
		21	0.008	0.009



Included observations: 4189

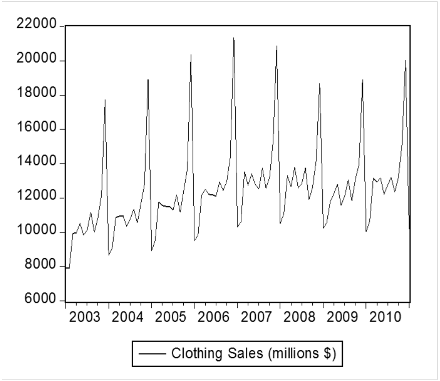
Autocorrelation	Partial Correlation		AC	PAC
		1	-0.056	-0.056
		2	-0.023	-0.027
		3	0.047	0.044
		4	-0.613	-0.612
		5	0.026	-0.040
		6	0.047	0.012
		7	-0.016	0.034
		8	0.134	-0.383
		9	-0.005	-0.031
		10	-0.034	0.002
		11	0.011	0.040
		12	-0.000	-0.229
		13	0.006	-0.010
		14	0.013	0.001
		15	-0.012	0.025
		16	-0.016	-0.155
		17	-0.019	-0.034
		18	-0.010	-0.020
		19	0.001	0.003
		20	0.003	-0.117
		21	0.022	-0.020

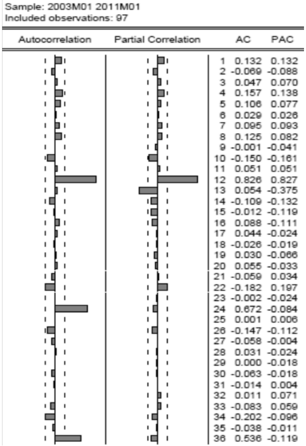
7.3.2 Seasonal ARMA Models

- AC and PAC functions of seasonal AR and MA models have similar characteristics as those of the non-seasonal AR and MA models, just occurring at multiples of s
- point forecast, forecast error, forecast uncertainty, and density forecast, can be also obtained in a similar way

7.3.2 Seasonal ARMA Models

- Monthly Clothing Sales in the United States, January 2003-January 2011, Figure07_17_clothingsales.xls





Sample: 2003M01 2011M01
Included observations: 97

	AC	PAC
1	0.132	0.132
2	-0.069	-0.088
3	0.047	0.070
4	0.157	0.138
5	0.106	0.077
6	0.029	0.026
7	0.095	0.093
8	0.125	0.082
9	-0.001	-0.041
10	-0.150	-0.161
11	0.051	0.051
12	0.826	0.827
13	0.054	-0.375
14	-0.109	-0.132
15	-0.012	-0.119
16	0.088	-0.111
17	0.044	-0.024
18	-0.026	-0.019
19	0.030	-0.066
20	0.055	-0.033
21	-0.059	0.034
22	-0.182	0.197
23	-0.002	-0.024
24	0.672	-0.084
25	0.001	0.006
26	-0.147	-0.112
27	-0.058	-0.004
28	0.031	-0.024
29	0.000	-0.018
30	-0.063	-0.018
31	-0.014	0.004
32	0.011	0.071
33	-0.083	0.059
34	-0.202	-0.096
35	-0.038	-0.011
36	0.536	-0.119

7.3.2 Seasonal ARMA Models

- seasonal component may also be a mixture of AR and MA dynamics - we define a general S-ARMA(P, Q) as

$$Y_t = c + \Phi_s Y_{t-s} + \Phi_{2s} Y_{t-2s} + \ldots + \Phi_{Ps} Y_{t-Ps} + \varepsilon_t + \Theta_s \varepsilon_{t-s} + \Theta_{2s} \varepsilon_{t-2s} + \ldots + \Theta_{Qs} \varepsilon_{t-Qs}$$

- equivalently, using lag operator, we can write S-ARMA(P, Q) as

$$(1 - \Phi_s L^s - \Phi_{2s} L^{2s} - \ldots - \Phi_{Ps} L^{Ps}) Y_t = c + (1 - \Theta_s L^s - \Theta_{2s} L^{2s} - \ldots - \Theta_{Qs} L^{Qs}) \varepsilon_t$$

7.3.2 Seasonal ARMA Models

- in practice, time series combine seasonal and nonseasonal components
- a very common modeling practice is to assume that both cycles interact with each other in a multiplicative fashion
- example: suppose that we have a quarterly time series and there are a seasonal cycle S-AR(2) and a nonseasonal cycle AR(1), the multiplicative model is written using lag operator as

$$(1 - \Phi_4 L^4 - \Phi_8 L^8)(1 - \phi_1 L)Y_t = c + \varepsilon_t$$

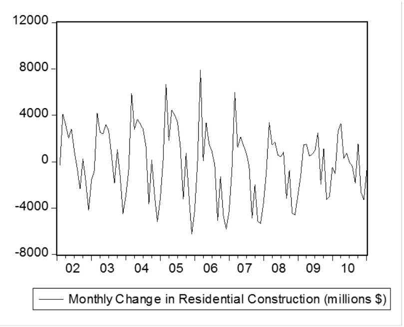
- example: suppose that we have a quarterly time series and there are a seasonal cycle S-ARMA(1,2) and a nonseasonal cycle ARMA(2,1), the multiplicative model is written using lag operator as

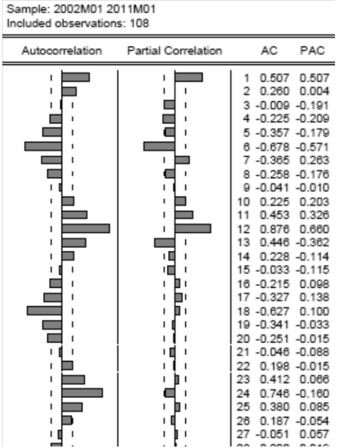
$$(1 - \Phi_4 L^4)(1 - \phi_1 L - \phi_2 L^2)Y_t = c + (1 - \Theta_4 L^4 - \Theta_8 L^8)(1 - \theta_1 L)\varepsilon_t$$

- in the multiplicative models the seasonal polynomials multiply the nonseasonal polynomials

7.3.2 Seasonal ARMA Models

- Monthly Changes in U.S. Residential Construction, January 2002-January 2011, Figure07_19_constructionchanges.xls





	AC	PAC
1	0.507	0.507
2	0.260	0.004
3	-0.009	-0.191
4	-0.225	-0.209
5	-0.357	-0.179
6	-0.678	-0.571
7	-0.365	0.263
8	-0.258	-0.176
9	-0.041	-0.010
10	0.225	0.203
11	0.453	0.326
12	0.876	0.660
13	0.446	-0.362
14	0.228	-0.114
15	-0.033	-0.115
16	-0.215	0.098
17	-0.327	0.138
18	-0.627	0.100
19	-0.341	-0.033
20	-0.251	-0.015
21	-0.046	-0.088
22	0.198	-0.015
23	0.412	0.066
24	0.746	-0.160
25	0.380	0.085
26	0.187	-0.054
27	-0.051	0.057

7.3.2 Seasonal ARMA Models

- Monthly Changes in U.S. Residential Construction, January 2002-January 2011
- based on AC and PAC we choose to estimate AR(1) + S-AR(1) Model

$$(1 - \Phi_{12} L^{12})(1 - \phi_1 L) Y_t = c + \varepsilon_t$$

- note that this is equivalent to an AR(13) specification

$$(1 - \phi_1 L - \Phi_{12} L^{12} - \phi_1 \Phi_{12} L^{13}) Y_t = c + \varepsilon_t$$

- in EViews in specification box enter **const c ar(1) sar(12)**

TABLE 7.3 Monthly Changes in Residential Construction, Estimation Results of AR(1) and S-AR(1) Model

Dependent Variable: change CONST
Method: Least Squares
Sample (adjusted): 2003M03 2011M01
Included observations: 95 after adjustments
Convergence achieved after 6 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-593.2408	2399.622	-0.247223	0.8053
AR(1)	0.439971	0.093551	4.703012	0.0000
SAR(12)	0.923569	0.038771	23.82102	0.0000

R-squared	0.894790	Mean dependent var	-128.3158
Adjusted R-squared	0.892502	S.D. dependent var	3036.076
S.E. of regression	995.4326	Akaike info criterion	16.67530
Sum squared resid	91161518	Schwarz criterion	16.75595
Log likelihood	-789.0768	F-statistic	391.2194
Durbin–Watson stat	2.115719	Prob(F-statistic)	0.000000

7.3.2 Seasonal ARMA Models

- Monthly Changes in U.S. Residential Construction, January 2002-January 2011
- multistep forecast from February 2011 to January 2012 with 95% confidence bands